Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number

Candidate Number

Pearson Edexcel International Advanced Level

Wednesday 31 May 2023

Morning (Time: 1 hour 30 minutes)

Paper reference **WMA13/01**

Mathematics

International Advanced Level
Pure Mathematics P3

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

1.

$$g(x) = x^6 + 2x - 1000$$

(a) Show that $g(x) = 0$ has a root α in the interval $[3, 4]$

(2)

Using the iteration formula

$$x_{n+1} = \sqrt[6]{1000 - 2x_n} \qquad \text{with } x_1 = 3$$

(b) (i) find, to 4 decimal places, the value of x_2

(ii) find, by repeated iteration, the value of α.
Give your answer to 4 decimal places.

(3)

1. a) $g(3) = -265$ given that $g(x)$ is continuous
$g(4) = 3104$ between 3 & 4
because there's a sign change
→ root lies between 3 & 4

b) (i) $x_{n+1} = \sqrt[6]{1000 - 2x_n}$

$x_1 = 3$

$x_2 = x_{1+1} = \sqrt[6]{1000 - 2(3)} = 3.1591$

(ii) $x_3 = 3.1589$ consistent to 4 d.p.

$x_4 = 3.1589$

$\alpha = 3.1589$

2.

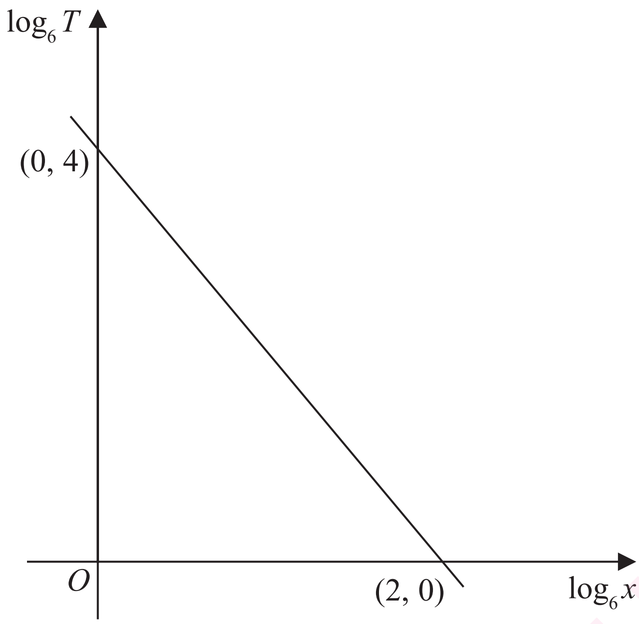


Figure 1

Figure 1 shows the linear relationship between $\log_6 T$ and $\log_6 x$

The line passes through the points (0, 4) and (2, 0) as shown.

(a) (i) Find an equation linking $\log_6 T$ and $\log_6 x$

(ii) Hence find the exact value of T when $x = 216$

(3)

(b) Find an equation, not involving logs, linking T with x

(3)

2. a) (i) Equation of line : $y - y_1 = m(x - x_1)$ coordinates of known point on line

gradient

Known points : $(0, 4)$ & $(2, 0)$

gradient $= \frac{\Delta y}{\Delta x} = \frac{0-4}{2-0} = -2$

$\log_6 T - 0 = -2(\log_6 x - 2)$

$\log_6 T = -2\log_6 x + 4$

P 7 2 8 7 0 A 0 4 3 2

Question 2 continued

(ii) when $x = 216$ →

$\log_6 T = -2\log_6(216) + 4$

$\log_6 T = -2$

$T = 6^{-2} = \frac{1}{6^2} = \frac{1}{36}$

LOG RULES → $\log_a b = c$ → $a^c = b$

b) LOG RULES → $a\log_b(c) = \log_b(c^a)$

$\log_a b + \log_a c = \log_a(bc)$

$\log_6 T = -2\log_6 x + 4$

$\log_6 T + 2\log_6 x = 4$

$\log_6 T + \log_6 x^2 = 4$

$\log_6(Tx^2) = 4$

$6^4 = Tx^2$ → $Tx^2 = 1296$

$T = \frac{1296}{x^2}$

(Total for Question 2 is 6 marks)

3. (i) Find $\frac{\mathrm{d}}{\mathrm{d}x}\ln\left(\sin^2 3x\right)$ writing your answer in simplest form.

(2)

(ii) (a) Find $\frac{\mathrm{d}}{\mathrm{d}x}\left(3x^2 - 4\right)^6$

(2)

(b) Hence show that

$$\int_0^{\sqrt{2}} x\left(3x^2 - 4\right)^5 \mathrm{d}x = R$$

where R is an integer to be found.

(*Solutions relying on calculator technology are not acceptable.*)

(3)

3. (i) let $y = \ln\left(\sin^2(3x)\right)$

$$\boxed{\text{CHAIN RULE}: \quad \frac{dy}{dx} = \frac{dy}{\cancel{du}} \times \frac{\cancel{du}}{dx}}$$

$u = \sin^2(3x)$ $\qquad \frac{du}{dx} = 2 \times \sin(3x) \times 3\cos(3x) = 6\sin(3x)\cos(3x)$

$y = \ln\left(\sin^2(3x)\right) = \ln(u)$ $\qquad \frac{dy}{du} = \frac{1}{u}$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= \frac{1}{u} \times 6\sin(3x)\cos(3x) = \frac{6\cancel{\sin(3x)}\cos(3x)}{\sin^2(3x)}$$

$$= 6\cot(3x)$$

(ii) a) let $y = (3x^2-4)^6$

$u = 3x^2 - 4$ $\qquad \frac{du}{dx} = 6x$

$y = (3x^2-4)^6 = u^6$ $\qquad \frac{dy}{du} = 6u^5$

Question 3 continued

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= 6u^5 \times 6x = 36(3x^2-4)^5 x$$

b) $\int_0^{\sqrt{2}} x(3x^2-4)^5 \, dx$

we know from (a) that

$\frac{d}{dx}(3x^2-4)^6 = 36x(3x^2-4)^5$

$\therefore \int 36x(3x^2-4)^5 \, dx = (3x^2-4)^6$

$$= \frac{1}{36}\int_0^{\sqrt{2}} 36x(3x^2-4)^5 \, dx$$

$$= \frac{1}{36}\left[(3x^2-4)^6\right]_0^{\sqrt{2}}$$

$$= \frac{1}{36}(64-4096) = -112$$

(Total for Question 3 is 7 marks)

4. The function f is defined by

$$f(x) = 2x^2 - 5 \qquad x \geqslant 0 \quad x \in \mathbb{R}$$

(a) State the range of f

(1)

On the following page there is a diagram, labelled Diagram 1, which shows a sketch of the curve with equation $y = f(x)$.

(b) On Diagram 1, sketch the curve with equation $y = f^{-1}(x)$.

(2)

The curve with equation $y = f(x)$ meets the curve with equation $y = f^{-1}(x)$ at the point P

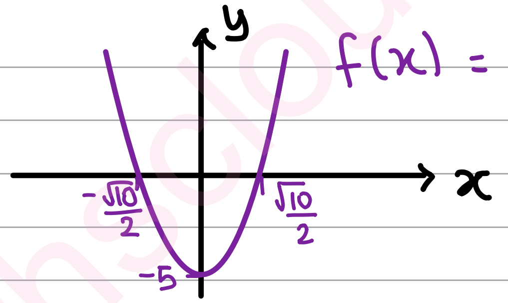
Using algebra and showing your working,

(c) find the exact x coordinate of P

(3)

4. a) $f(x) = 2x^2 - 5$

$x \geqslant 0$, $x \in R$



$\therefore f(x) \geqslant -5$

b)

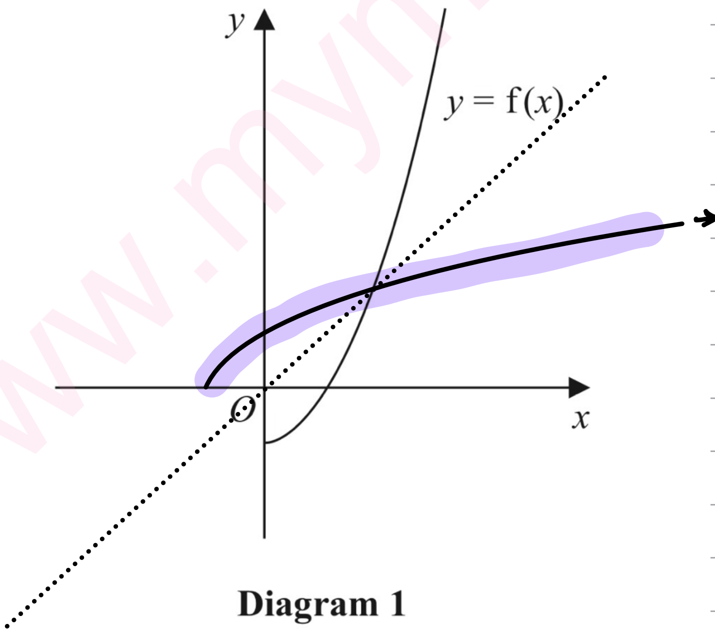


Diagram 1

inverse functions are the reflection of the function in the line $y=x$

$\rightarrow f^{-1}(x)$ is inverse of $f(x)$

$\therefore$ reflection of $f(x)$ in $y=x$

Question 4 continued

c) $f(x)$ meets $f^{-1}(x)$ on the line $y=x$

$\therefore$ P is when $f(x) = x$

$$2x^2 - 5 = x$$

$$2x^2 - x - 5 = 0$$

$$x = \frac{1 \pm \sqrt{41}}{4}$$

reject negative solution as $x \geqslant 0$

$\therefore$ P : $x = \dfrac{1+\sqrt{41}}{4}$

5. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 < x < \pi$

$$(x - 2)\left(\sqrt{3}\sec x + 2\right) = 0$$

(3)

(ii) Solve, for $0 < \theta < 360°$

$$10\sin\theta = 3\cos 2\theta$$

(4)

5.(i) $(x-2)(\sqrt{3}\sec(x)+2) = 0$ $\quad 0 < x < \pi$

$x = 2$ $\cup$ $\sec(x) = -\dfrac{2}{\sqrt{3}}$

$\cos(x) = -\dfrac{\sqrt{3}}{2}$ $\quad x = \dfrac{5\pi}{6}$

(ii) $10\sin(\theta) = 3\cos(2\theta)$

USING DOUBLE ANGLE FORMULA:

$\cos(2A) = \cos^2(A) - \sin^2(A)$

$= (1-\sin^2(A)) - \sin^2(A)$

$= 1 - 2\sin^2(A)$

$10\sin(\theta) = 3(1-2\sin^2(\theta))$

$6\sin^2(\theta) + 10\sin(\theta) - 3 = 0$

let $u = \sin(\theta)$

$6u^2 + 10u - 3 = 0$

$u = \sin(\theta) = \dfrac{-5 \pm \sqrt{43}}{6}$

reject $\sin(\theta) > 1$ as $-1 \leq \sin(\theta) \leq 1$ range

$\theta = 15.0°$ $\cup$ $165°$

6.

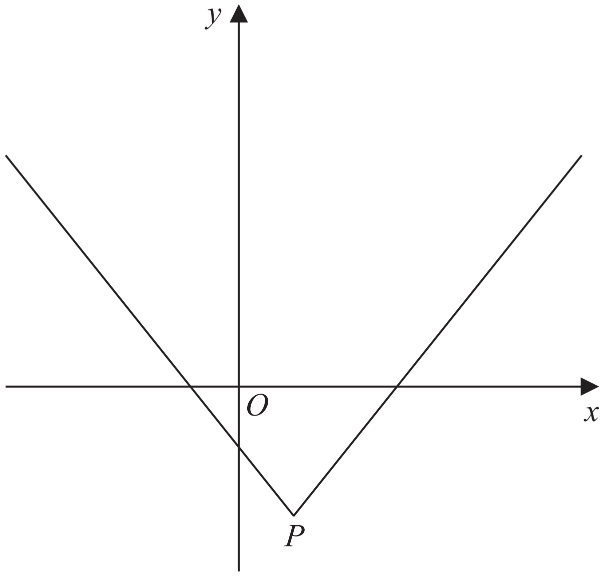


Figure 2

Figure 2 shows a sketch of the graph $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = 3|x - 2| - 10$$

The vertex of the graph is at point P, shown in Figure 2.

(a) Find the coordinates of P **(2)**

(b) Find $\mathrm{ff}(0)$ **(2)**

(c) Solve the inequality

$$3|x - 2| - 10 < 5x + 10$$

(2)

(d) Solve the equation

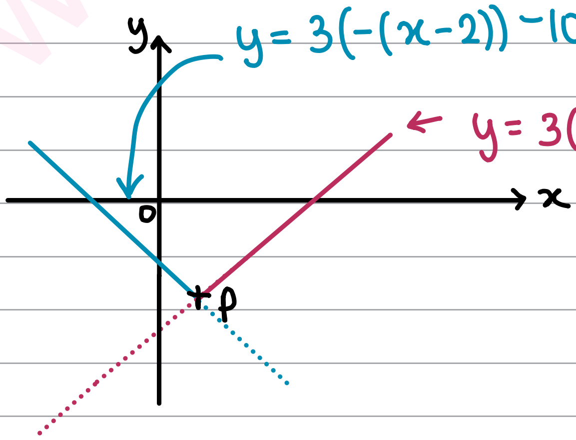
$$\mathrm{f}(|x|) = 0$$

(3)

6. a)

$y = 3(-(x-2)) - 10 = -3x - 4$

$\leftarrow y = 3(x-2) - 10 = 3x - 16$



Question 6 continued

P is where the 2 lines meet

$-3x-4 = 3x-16$

$6x = 12$

$x = 2$ P: $(2, -10)$

b) $f(f(0))$

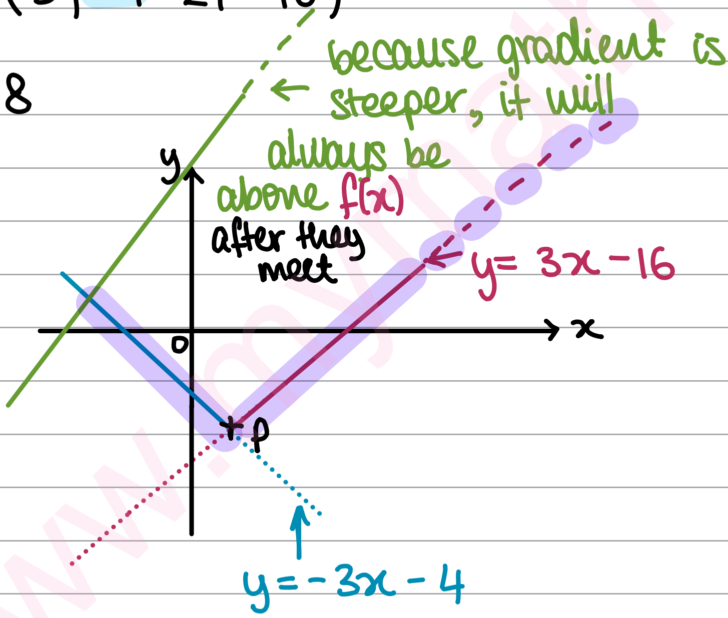
$= f(3|0-2|-10)$

$= f(-4)$

$= (3|-4-2|-10)$

$= 8$

c)



$3|x-2|-10 < 5x+10$

Point X where $y = 5x+10$ meets $y=-3x-4$

$5x+10 = -3x-4$

$8x = -14$ $x = -\frac{7}{4}$

Turn over

P 7 2 8 7 0 A 0 1 5 3 2

Question 6 continued

$\therefore\ 3|x-2|-10 < 5x+10$ when $x > -\frac{7}{4}$

d) METHOD 1 (looking at both sections of modulus):

$f(|x|) = 0$

$y = -3|x| - 4 = 0$

$-3|x| = 4$

$\cancel{|x| = -\frac{4}{3}}$

reject as

$|x|$ must be ≥ 0

$y = 3|x| - 16 = 0$

$3|x| = 16$

$|x| = \frac{16}{3}$

$\therefore\ |x| = \frac{16}{3}$

$x = \pm\frac{16}{3}$

METHOD 2 (substitute into equation):

$f(|x|) = 3\big||x|-2\big| - 10 = 0$

$\big||x|-2\big| = \frac{10}{3}$

If $|x|-2 > 0 \rightarrow |x| > 2$

$|x| - 2 = \frac{10}{3}$

$|x| = \frac{16}{3}$

$|x|-2 < 0 \rightarrow |x| < 2$

$|x|-2 = -\frac{10}{3}$

$\cancel{|x| = -\frac{4}{3}}$

reject as

$|x|$ must be ≥ 0

$\therefore\ |x| = \frac{16}{3}$ $\qquad x = \pm\frac{16}{3}$

7. A scientist is studying two different populations of bacteria.

The number of bacteria N in the first population is modelled by the equation

$$N = Ae^{kt} \qquad t \geqslant 0$$

where A and k are positive constants and t is the time in hours from the start of the study.

Given that

- there were 2500 bacteria in this population at the start of the study
- there were 10 000 bacteria 8 hours later

(a) find the exact value of A and the value of k to 4 significant figures.

(3)

The number of bacteria N in the second population is modelled by the equation

$$N = 60\,000e^{-0.6t} \qquad t \geqslant 0$$

where t is the time in hours from the start of the study.

(b) Find the rate of decrease of bacteria in this population exactly 5 hours from the start of the study. Give your answer to 3 significant figures.

(2)

When $t = T$, the number of bacteria in the two different populations was the same.

(c) Find the value of T, giving your answer to 3 significant figures.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(3)

7. a) $N = Ae^{kt} \qquad t \geq 0$

when $t = 0$

$\hookrightarrow Ae^{k(0)} = A = 2500$

when $t = 8$

$\hookrightarrow Ae^{k(8)} = 25\cancel{00}e^{8k} = 100\cancel{00}$

$e^{8k} = 4$

$8k = \ln(4)$

$k = \dfrac{\ln(4)}{8} = 0.1733$

P 7 2 8 7 0 A 0 1 8 3 2

Question 7 continued

b) rate of decrease of bacteria $= -\dfrac{dN}{dt}$

$N = 60000\, e^{-0.6t}$

$\dfrac{d}{dx}(e^{kx}) = ke^{kx}$

$\dfrac{dN}{dt} = 60000 \times -0.6 \times e^{-0.6t}$

when $t = 5 \rightarrow -\dfrac{dN}{dt} = 60000 \times -0.6 \times e^{-0.6(5)}$

$= -(-1792)$

$\therefore$ rate of decrease $= 1790$

c) when $t = T$

$25\cancel{00}\, e^{kT} = 600\cancel{00}\, e^{-0.6T}$

$\dfrac{e^{kT}}{e^{-0.6T}} = 24$

$e^{T(k+0.6)} = 24$

$T(k+0.6) = \ln(24)$

$T = \dfrac{\ln(24)}{\frac{\ln(4)}{8} + 0.6} = 4.11$

8.

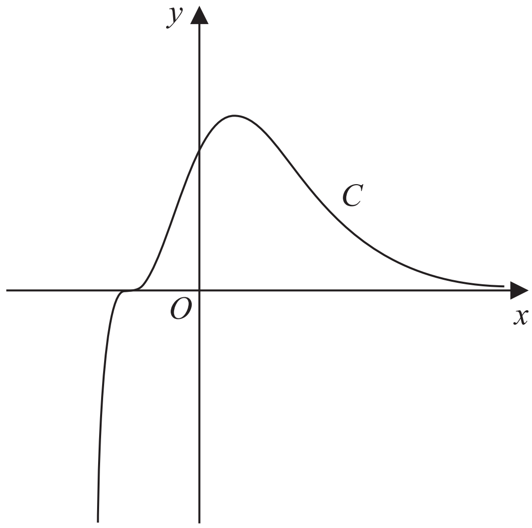


Figure 3

Figure 3 shows a sketch of the curve C with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = (2x + 1)^3 e^{-4x}$$

(a) Show that

$$\mathrm{f}'(x) = A(2x + 1)^2 (1 - 4x)\, e^{-4x}$$

where A is a constant to be found.

(4)

(b) Hence find the exact coordinates of the two stationary points on C.

(3)

The function g is defined by

$$\mathrm{g}(x) = 8\,\mathrm{f}(x - 2)$$

(c) Find the coordinates of the maximum stationary point on the curve with equation $y = \mathrm{g}(x)$.

(2)

8. a) $f(x) = (2x+1)^3 e^{-4x}$

$\boxed{\text{PRODUCT RULE : } y = uv \quad y' = u'v + uv'}$

$u = (2x+1)^3$ $\qquad \frac{du}{dx} = 3 \times 2 \times (2x+1)^2$

$v = e^{-4x}$ $\qquad \frac{dv}{dx} = -4e^{-4x}$

Question 8 continued

$$f'(x) = \left(6(2x+1)^2\right)\left(e^{-4x}\right) + \left((2x+1)^3\right)\left(-4e^{-4x}\right)$$

$$= e^{-4x}(2x+1)^2\left(6 + (2x+1)(-4)\right)$$

$$= e^{-4x}(2x+1)^2(-8x+2)$$

$$= 2(2x+1)^2(1-4x)e^{-4x} \quad \rightarrow A=2$$

b) stationary points $\rightarrow \frac{dy}{dx} = 0$

$$2(2x+1)^2(1-4x)e^{-4x} = 0$$

$$2x+1=0 \quad \cup \quad 1-4x=0$$

$$x = -\frac{1}{2} \quad \cup \quad x = \frac{1}{4}$$

$$\therefore \left(-\frac{1}{2}, 0\right), \left(\frac{1}{4}, \frac{27}{8e}\right)$$

c) $g(x) = 8f(x-2)$

stretch : scale factor 8
parallel to y-axis

translation through the vector: $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$

max $f(x) \rightarrow \left(\frac{1}{4}, \frac{27}{8e}\right)$

$$\left(\frac{1}{4}, \frac{27}{8e}\right) \xrightarrow[\text{stretch}]{} \left(\frac{1}{4}, \frac{27}{8e}\times 8\right) \xrightarrow[\text{translation}]{} \left(\frac{1}{4}+2, \frac{27}{e}\right) = \left(\frac{9}{4}, \frac{27}{e}\right)$$

9. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\frac{\cos 2x}{\sin x} + \frac{\sin 2x}{\cos x} \equiv \operatorname{cosec} x \qquad x \neq \frac{n\pi}{2} \qquad n \in \mathbb{Z}$$

(3)

(b) Hence solve, for $0 < \theta < \frac{\pi}{2}$

$$\left(\frac{\cos 2\theta}{\sin\theta} + \frac{\sin 2\theta}{\cos\theta}\right)^2 = 6\cot\theta - 4$$

giving your answers to 3 significant figures as appropriate.

(5)

(c) Using the result from part (a), or otherwise, find the exact value of

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \left(\frac{\cos 2x}{\sin x} + \frac{\sin 2x}{\cos x}\right)\cot x \, dx$$

(2)

9.a) LHS : $\dfrac{\cos(2x)}{\sin(x)} + \dfrac{\sin(2x)}{\cos(x)}$ RHS : $\operatorname{cosec}(x)$

$= \dfrac{1}{\sin(x)}$

USING COMPOUND ANGLE FORMULAE $\rightarrow$ $\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)$

$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$

$= \dfrac{\cos^2(x) - \sin^2(x)}{\sin(x)} + \dfrac{2\sin(x)\cancel{\cos(x)}}{\cancel{\cos(x)}}$

$= \dfrac{\cos^2(x) - \sin^2(x) + 2\sin^2(x)}{\sin(x)}$

$= \dfrac{\cos^2(x) + \sin^2(x)}{\sin(x)} \leftarrow \boxed{\sin^2 A + \cos^2 A = 1}$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 9 continued

$= \frac{1}{\sin(x)} = RHS$

b) $0 < \theta < \frac{\pi}{2}$

$\left(\frac{\cos(2\theta)}{\sin(\theta)} + \frac{\sin(2\theta)}{\cos(\theta)}\right)^2 = \left(\frac{1}{\sin(x)}\right)^2$

$= \operatorname{cosec}^2(\theta) = 6\cot(\theta) - 4$

$\sin^2 A + \cos^2 A = 1$ ← divide through by $\sin^2 A$

$\frac{\sin^2 A}{\sin^2 A} + \frac{\cos^2 A}{\sin^2 A} = \frac{1}{\sin^2 A}$

$\boxed{1 + \cot^2(A) = \operatorname{cosec}^2(A)}$

$1 + \cot^2(\theta) = 6\cot(\theta) - 4$

$\cot^2(\theta) - 6\cot(\theta) + 5 = 0$

$(\cot(\theta) - 5)(\cot(\theta) - 1) = 0$

$\cot(\theta) = 5 \cup 1$

$\tan(\theta) = \frac{1}{5} \cup 1$

$\theta = 0.197 \cup \frac{\pi}{4}$

Question 9 continued

(c) $\displaystyle\int_{\pi/6}^{\pi/4} \left(\frac{\cos(2x)}{\sin(x)} + \frac{\sin(2x)}{\cos(x)} \right) \cot(x) \; dx$

$= \displaystyle\int_{\pi/6}^{\pi/4} \operatorname{cosec}(x)\cot(x) \; dx$

$= \left[-\operatorname{cosec}(x) \right]_{\pi/6}^{\pi/4}$

$= 2 - \sqrt{2}$

10.

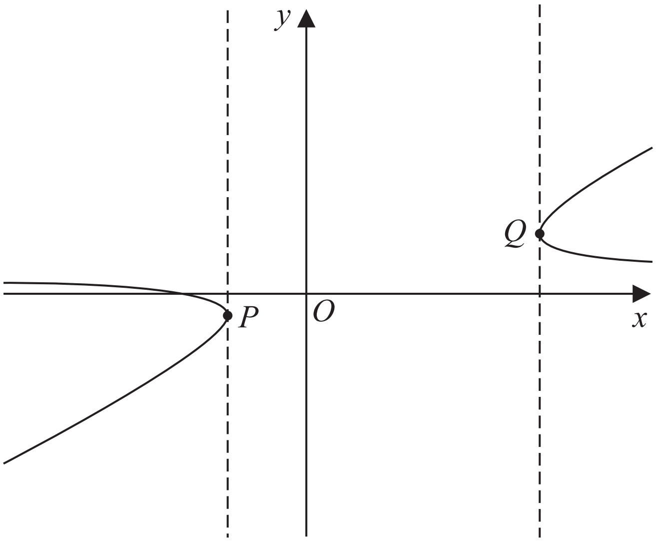


Figure 4

Figure 4 shows a sketch of the curve with equation

$$x = \frac{2y^2 + 6}{3y - 3}$$

(a) Find $\frac{dx}{dy}$ giving your answer as a fully simplified fraction.

(4)

The tangents at points P and Q on the curve are parallel to the y-axis, as shown in Figure 4.

(b) Use the answer to part (a) to find the equations of these two tangents.

(4)

10. a) $x = \frac{2y^2+6}{3y-3}$

Quotient rule for differentiating : $y = \frac{u}{v} \rightarrow \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$u = 2y^2 + 6$ $\qquad \frac{du}{dy} = 4y$

$v = 3y - 3$ $\qquad \frac{dv}{dy} = 3$

$\frac{dx}{dy} = \frac{(3y-3)(4y) - (2y^2+6)(3)}{(3y-3)^2} = \underline{12y^2 - 12y - 6y^2 - 18}$

Question 10 continued

$$= \frac{6y^2-12y-18}{9(y-1)^2} = \frac{2y^2-4y-6}{3(y-1)^2}$$

b) tangents at P & Q is when $\frac{dx}{dy} = 0$

$$\frac{2y^2-4y-6}{3(y-1)^2} = 0$$

$$2y^2-4y-6=0$$

$$(2y-6)(y+1) = 0$$

$y = 3 \cup -1$

$\downarrow \qquad \downarrow$

$x=4 \qquad x=-\frac{4}{3}$

$\therefore\ P = \left(-\frac{4}{3}, -1\right) \qquad Q = (4,3)$

so $\text{tangent}_P \rightarrow x = -\frac{4}{3} \qquad \text{tangent}_Q \rightarrow x = 4$